Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (OLD) EXAMINATION – SUMMER 2021

Subject Code:140001 **Date:03/09/2021**

Subject Name:Mathematics-IV

Time:02:30 PM TO 05:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 **(a)** Find real and imaginary parts of $(-1-i)^7+(-1+i)^7$. **07**

(b) (i) Solve the equation $z^2-(5+i)z+8+i=0$. **04**

(ii) Find the modulus and amplitude of i^i. **03**

Q.2 **(a)** Prove that an analytic function with constant modulus is constant **07**

(b) Find and sketch the image of region $x\geq 1$ under the transformation $w=\frac{1}{z}$. **07**

OR

(b) Define the mobius transformation and determine the mobius transformation that maps $z_1=0, z_2=1, z_3=\infty$ onto $w_1=-1, w_2=-i, w_3=1$ respectively. **07**

Q.3 **(a)** Evaluate $\int_C (x^2-iy^2)dz$ along the parabola $y=2x^2$ from $(1,2)$ to $(2,8)$. **07**

(b) Evaluate $\oint_C \frac{dz}{z^2+1}$, where C is $|z+i|=1$ in counterclockwise. **07**

OR

Q.3 **(a)** Write the two Laurent's series expansion in powers of z that represent the function $f(z)=\frac{1}{z^2(1-z)}$ in certain domains and also specify domains. **07**

(b) Find a real root of the equation $x^3-4x-9=0$, using the bisection method in four stages up to 3 decimal places. **07**

Q.4 **(a)** Find a root of $x^4-x^3+10x+7=0$ correct to three decimal places between a= -2 and b = - 1by using Newton-Raphson method. **07**

(b) Find the largest eigen value for $A=\begin{bmatrix}4 & 4 & 2\\ 4 & 4 & 1\\ 2 & 1 & 8\end{bmatrix}$. **07**

OR

Q.4 **(a)** Solve $2x+5y-3z=1, 5x+y+4z=2$ *and* $7x+3y+z=4$ by using Gauss-Jordan method. **07**

(b) Solve $2x+y+54z=110, 27x+6y-z=85$ *and* $6x+15y+2z=72$ correct upto three decimal places by using Gauss-Seidel method. **07**

Q.5 **(a)** For $\frac{dy}{dx}=y-\frac{2x}{y}, y(0)=1, h=0.1$ find y(0.2) by using Euler's method. **07**

(b) Compute cosh(0.56) using Newton's forward difference formula and also estimate the error for the following table. **07**

x	0.5	0.6	0.7	0.8
f(x)	1.127626	1.185465	1.255169	1.337435

OR

Q.5 **(a)** Find the Lagrange's interpolating polynomial from the following data **07**

x	0	1	4	5
f(x)	1	3	24	39

(b) Evaluate $\int_0^6 \frac{1}{1+x}dx$, taking h = 1 and using Simpson's $\frac{1}{3}$ rule. Hence obtain an approximate value of $\log_e 7$. **07**

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